Music's "DNA" - The Perfect 5th

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In this article, Marianne describes a few of the many ways in which the interval of the perfect 5th underlies so many patterns and forms found in scales and modes. She explores how understanding these patterns can improve comprehension and memory and provide a source of inspiration and delight to those interested in the intricacies of music. Marianne's novel framing of and perspectives on the structure of the diatonic scale reveal patterns, form, and order to what might seem on the surface to be random. The ideas outlined in the article below have been an essential part of her teaching since 1991.

The Origins of the Diatonic Scale

When we look at the musical keyboard, we view a series of equally spaced white notes that might lead us to believe that the pitch space between adjacent white notes is a whole step in each case. This, however, is not the case because there is a half step between both E and F and B and C. Importantly, while most of us might intuitively sing a diatonic scale accurately, we probably have no conscious knowledge of the location of whole and half steps. This fact is significant because few of us would accidentally sing a scale without these half steps in the correct place. This pattern seems somehow 'right', though it is difficult to explain why. Therefore we must wonder why these two half steps exist where they exist in the context of the five other whole steps in the scale.

The seven-note diatonic scale is thought to have existed since the time of Pythagoras, 500 B.C.E. Musicologists have long asserted that it was Pythagoras who discovered the importance of the interval of a perfect 5th in the creation of the scale. It was Pythagoras who, in the West, discovered that the familiar scale could be derived by dividing a string into two-thirds its length to obtain a pitch a perfect fifth higher, repeating this process six times until seven diatonic pitches are obtained. So, if a string pitched at F is thus divided to a 2:3 ratio along its length, two-thirds of that string will sound C. Divide this C string similarly, then two-thirds its length will produce the pitch G. Divide this string likewise and two-thirds its length will result in the pitch D, et cetera, until the note B is eventually reached.

The reader might wonder why I have used F as the first note rather than C. Were we to start on C, we would need an accidental - F-sharp - as the seventh note, a perfect 5th above B. By starting a 5th lower than C, we obtain each of the seven notes of the diatonic scale without an accidental. Furthermore, the notes follow the pattern of sharps as they appear in the key signatures. Starting at the end of the list and reading backwards gives you the order of flats.

String Order Number	1	2	3	4	5	6	7
Note Correspondence	F	C	G	D	A	E	B

From this, this pattern of fifths does not resemble a scale until we notice the pattern: every other note is the adjacent note in the scale. That is, from F we skip over C to get G, next we skip over D to get A, then we skip over E to get B, then we double back, skipping over F to C, then we skip over G to get D, then skip over A to get E then, to complete the octave scale, from E, skip over B and circle back to where we began on F.

String Order Number	1	3	5	7	2	4	6
Note Correspondence	F	G	A	B	C	D	E

From this, we obtain the diatonic scale starting on F, consisting of whole steps between adjacent notes excepting between B and C and between E and F. Similarly, by starting on C and following the same procedure, the C major scale is created.

String Order Number	2	4	6	1	3	5	7
Note Correspondence	C	D	E	F	G	A	B

The various diatonic seven-note modes are created by starting on a different one of the seven notes, obtaining each note in diatonic order by naming every-other-note according to the pattern, which will be discussed in detail shortly.

The 12-note Chromatic Octave from Perfect Fifths

Pythagoras also supposedly discovered the circle-of-fifths by continuing to divide the string into two-thirds until he obtained five more notes beyond the seven, arriving after twelve divisions at approximately the same pitch as the first. A discrepancy, referred to as the

Pythagorean Comma, existed between the first and last pitches, such that a more accurate term for this series of 13 pitches might better be called the “spiral” rather than the “circle”-of-fifths. In the Western musical tradition, reconciling the Pythagorean Comma has been a matter of hiding, disguising or sequestering the discrepancy somewhere among the twelve-note collection. Thus, the 12-note chromatic octave has its roots in this underlying pattern of pitches a perfect fifth apart.

String Order Number	1	2	3	4	5	6	7	8	9	10	11	12	13
Note Correspondence	F	C	G	D	A	E	B	F#	C#	G#	D#	A#	E#

String Order Number	1	8	3	10	5	12	7	2	9	4	11	6	13
Note Correspondence	F	F#	G	G#	A	A#	B	C	C#	D	D#	E	E#

Observation: The number “7” and the number “5” recur often in musical patterns. For example, there are 7 diatonic notes in the scale built upon 5ths; perfect fifths consists of 7 half steps; 7 plus 5 equals 12; and the number order for each consecutive one of the 12 notes in the chromatic octave, listed above, is achieved by adding 7 to the preceding number in a BASE-12 system (1 + 7 = 8, 8 + 7 = 3, 3 + 7 = 10, et cetera).

The Pentatonic Scale

According to Alain Danielou in his book Music and the Power of Sound, it was the Chinese who first discovered the notes of the scale by the systematic division into two-thirds, in order to form perfect fifths, at least 500 years before Pythagoras. The difference between the Greeks and the Chinese was that the Chinese were not disturbed by the discrepancy between the 1st and 13th note in the series, considering the 13th note to be a wholly different note of its own and thereby allowing for a continuation of the cycle. The Chinese system theoretically renders more than 12 notes to the octave, but such a scale would not be practically employed. Instead, the Chinese favored using only five consecutive notes fifths apart to form the pentatonic scale.

String Order Number	1	2	3	4	5
Note Correspondence	C	G	D	A	E

Arranged diatonically, the notes in the pentatonic scale will be formed, as before, by arranging the every-other-note in order. Again, various forms of the same scale can be obtained by starting on a different one of the five notes.

String Order Number	1	3	5	2	4
Note Correspondence	C	D	E	G	A

String Order Number	3	5	2	4	1
Note Correspondence	D	E	G	A	C

String Order Number	5	2	4	1	3
Note Correspondence	E	G	A	C	D

String Order Number	2	4	1	3	5
Note Correspondence	G	A	C	D	E

String Order Number	4	1	3	5	2
Note Correspondence	A	C	D	E	G

Observation: Each division order number above is obtained by adding two in a base-five system; 1 + 2 = 3; 3 + 2 = 5; 5 + 2 = 2; 2 + 2 = 4; 4 + 2 = 1.

According to Danielou, Chinese musicians chose the five-note scale over the seven-note scale because of the instability from the diminished fifth - the only non-perfect fifth - that existed between the first and last notes in the scale. It can be seen that the interval from either B up to F or F up to B is [6], not [7] half steps.

F C G D A E **B**

The pentatonic scale results when the outside two notes in the seven-note scale are eliminated. Thus, the pentatonic scale, among the most prevalently-employed scales throughout the globe, avoids the single fifth that is not perfect.

Further, while the Chinese octave can contain many more notes because the 13th and subsequent notes are not the same as the ones preceding, there is a decided preference for employing only five consecutive, purely tuned notes to create the scale, not 13 or more.

The Diatonic Modes

According to the 17th-century theorist Gioseffo Zarlino, there were 6 principal diatonic modes formed on each of the seven diatonic notes excepting B, with each of these 6 possessing a hypo version that started and ended on the dominant (degree 5). Zarlino's modes are listed in order below. The degrees of the scale, 1 through 7, are listed in the top row, each number being accompanied by the sign ˆ above, according to modern conventions. Strong notes within each mode are indicated by bold typeface; the note of primary stress is colored blue, and the note of secondary stress is colored green.

<u>Zarlino's Order of Modes using note names</u>

**Zarlino named the modes by number only. Below, modes' names follow the church conventions.*

	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$
1st: Dorian	**D**	E	F	G	**A**	B	C	**D**
2nd: Hypodorian	**A**	B	C	**D**	E	F	G	**A**
3rd: Phrygian	**E**	F	G	A	**B**	C	D	**E**
4th: Hypophrygian	**B**	C	D	**E**	F	G	A	**B**
5th: Lydian	**F**	G	A	B	**C**	D	E	**F**
6th: Hypolydian	**C**	D	E	**F**	G	A	B	**C**
7th: Mixolydian	**G**	A	B	C	**D**	E	F	**G**
8th: Hypomixolydian	**D**	E	F	**G**	A	B	C	**D**
9th: Aeolian	**A**	B	C	D	**E**	F	G	**A**
10th: Hypoaeolian	**E**	F	G	**A**	B	C	D	**E**
11th: Ionian	**C**	D	E	F	**G**	A	B	**C**
12th: Hypoionian	**G**	A	B	**C**	D	E	F	**G**

*blue indicates the primary note of stress; green indicates the secondary note of stress

From the list above, we might wonder at the distinction between the 2nd and 9th modes, between the 3rd and 10th modes, between the 6th and the 11th modes, between the 8th and the 1st modes, between the 10th and the 3rd modes and between the 12th and the 7th modes, because the notes are the same in these pairs of modes. The distinction between these modes relates to the secondary note of stress in each mode. In each of the principal modes, the secondary note of stress after the 1st degree is the 5th; in each of the hypo modes, the secondary note of stress after the 1st degree is the 4th. Whereas the principal modes have a tonic-dominant-tonic profile, the hypo modes begin and end on the dominant of the preceding mode and possess a dominant-tonic-dominant profile, so that the strongest notes are the same in the principal and hypo versions. Importantly, each hypo mode is formed upon the note class a perfect fifth higher than the principal mode.

The mode formed on the note B was omitted by Zarlino and his predecessors because of the interval of a diminished fifth formed between degrees 1 and 5 of the scale - the strongest degrees. No mode having this interval between its tonic and dominant degrees could be permitted, because early theorists referred to the eerie-sounding diminished 5th as the "diabolo in musica" - the devil in music. Though this interval exists in each of the other modes, it does not occur in conjunction with the tonic of the mode.

Following church tradition, Zarlino identifies twelve modes, each consisting of seven diatonic notes, each defined by the first note (which is also the last) and the next most important note that can be either a fourth or a fifth higher or lower. Thus, there could be a different mode formed above or below the notes D, E, F, G, A and C, each secondarily emphasizing a melody with a tonic-dominant-tonic octave profile or, in the case of the six hypo modes, a dominant-tonic-dominant octave profile. In the ordering suggested by Zarlino, the first mode begins on what is commonly know as the Dorian. Excepting for the obvious diatonic ordering, it is difficult to ascertain why the hypo modes are included after the principal form of the mode until we look more deeply at the intervals formed between the 1st degree and each of the other six degrees. Below, I list the Zarlino's order of the modes, this time identifying the notes according to their primary di-chord number, that is, the number of half steps between each degree and the tonic. This is what I did to understand the modes more fully from the standpoint of the unique sound properties associated with each.

Zarlino's Order of Modes using Di-chord Numbers

	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$
1st: Dorian	0	2	3	5	7	9	10	0
2nd: Hypodorian	0	2	3	5	7	8	10	0
3rd: Phrygian	0	1	3	5	7	8	10	0
4th: Hypophrygian	0	1	3	5	6	8	10	0
5th: Lydian	0	2	4	6	7	9	11	0
6th: Hypolydian	0	2	4	5	7	9	11	0
7th: Mixolydian	0	2	4	5	7	9	10	0
8th: Hypomixolydian	0	2	3	5	7	9	10	0
9th: Aeolian	0	2	3	5	7	8	10	0
10th: Hypoaeolian	0	1	3	5	7	8	10	0
11th: Ionian	0	2	4	5	7	9	11	0
12th: Hypoionian	0	2	4	5	7	9	10	0

Observations: First, by following Zarlino's ordering, including the hypo modes, only one interval differs between neighboring modes, excepting between the 4th and 5th and between the 10th and 11th modes. We must remember that the hypo version for each mode begins and ends on the note a perfect fifth higher. However, in the case of the 4th and 5th modes, there are NO common intervals; in the case of the 10th and 11th modes, this is where Zarlino skipped over the Locrian. Had he not done this, the pattern would have remained linear between the 10th and the Locrian and between the Hypolocrian and the 11th mode, as can be seen below, however there would be NO shared intervals between the Locrian and the Hypolocrian, with the novel exception that both of these modes are the only ones that contain (6).

	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$
10th: Hypoaeolian	0	1	3	5	7	8	10	0
x1: Locrian	0	1	3	5	6	8	10	0
x2: Hypolocrian	0	2	4	6	7	9	11	0
11th: Ionian	0	2	4	5	7	9	11	0

An explanation for the extraordinary divergence between x1 and x2 is the fact that the first of these begins on the note B and the other on the note F - the outside two note elements of the seven-note scale, expressed as unstable by the Chinese.

When ordering the modes in the conventional modern way, where there are no hypo modes and where the first mode is on C, the 2nd on D, the 3rd on E, continuing in diatonic order, this pattern of relationship between modes is completely lost, as can be seen below. It was here that my studies began, as this was the ordering espoused in most theory texts and classes. By assigning each note an interval number, I could see that diatonically adjacent modes have at least two intervals that differ, making it difficult to see any easily discernible pattern. Without a discernible pattern, it would be very difficult to memorize the intervals in each of the modes.

Diatonic Ordering of the Modes

	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$
1st: Ionian	0	2	4	5	7	9	11	0
2nd: Dorian	0	2	3	5	7	9	10	0
3rd: Phrygian	0	1	3	5	7	8	10	0
4th: Lydian	0	2	4	6	7	9	11	0
5th: Mixolydian	0	2	4	5	7	9	10	0
6th: Aeolian	0	2	3	5	7	8	10	0
7th: Locrian	0	1	3	5	6	8	10	0

By looking more deeply, I would eventually see the emergence of a pattern. I would start by observing that the degrees that differ between diatonic neighboring modes were often a fifth apart from one another, for instance, the degrees that differ between the Ionian and Dorian modes are the 3rd and 7th degrees - degrees that are a perfect fifth apart; the degrees that differ between the Dorian and Phrygian are the 2nd and 6th that are, again, a perfect fifth apart. However, this pattern of the two differing degrees being a fifth apart was broken in the case of the Phrygian and Lydian modes, where there are five intervals that differ. The pattern of the two differing degrees being a fifth apart was once again resumed between the Lydian and Mixolydian, between the Mixolydian and Aeolian and even between the Aeolian and Locrian.

It then occurred to me to observe what might happen if I arranged the modes in order of perfect 5ths, starting with the mode beginning on F, the next on C, the next on G, the next on D, the next on A, the next on E and the last on B. In doing this, I observed that only one interval would differ between neighbors.

Ploger Ordering of the Modes by 5ths based on F

	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$
1st: Lydian	0	2	4	6	7	9	11	0
2nd: Ionian	0	2	4	5	7	9	11	0
3rd: Mixolydian	0	2	4	5	7	9	10	0
4th: Dorian	0	2	3	5	7	9	10	0
5th: Aeolian	0	2	3	5	7	8	10	0
6th: Phrygian	0	1	3	5	7	8	10	0
7th: Locrian	0	1	3	5	6	8	10	0

Degrees of close relationship

The figure above is, I maintain, the most natural ordering of the modes because it reveals a clear ordering of the degrees of similarity and differences, the modes nearest to one another sharing the most notes and those farthest sharing the fewest notes. In the ordering of the 5ths above, the farther apart the two modes are from one another, the more dissimilar they are in their primary di-chord composition. Therefore, the Lydian and the Locrian are the most dissimilar, sharing only that they alone contain the primary di-chord [6] - the augmented 4th/diminished 7th. It is worth noting that these two extreme modes have as their respective tonic, note classes a di-chord [6] apart.

	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$
Ploger-1st: Lydian	0	2	4	6	7	9	11	0
Ploger-7th: Locrian	0	1	3	5	6	8	10	0

Strong and Weak Modes

As a student of Nadia Boulanger, I was required to thoroughly study Theodore Dubois' "Traité d'harmonie," in which he suggests that chords of the major scale be grouped into strong (I, IV, V) or weak (iii, vi, vii°) categories. While most theorists would not argue with this proposal, it is not always apparent why one group is strong and the other weak, unless it is because the strong group is comprised of major triads whereas the weak group is comprised two minor and one diminished triad. This would not explain why, in the minor mode, $\hat{1}$ and $\hat{4}$ are considered strong degrees though they are both the roots of minor triads. Since at least the publishing of Rameau's 1728 "Treatise on Harmony," it has been commonly understood that the tonic and the notes a perfect 5th higher and lower are of greater importance than all other note degrees of the scale, and that the dominant (V) and sub-dominant (IV) chords are essential in the establishment of a key. Rameau reasoned that the ear understands that the essential 3:2 ratio (the perfect 5th) is responsible for our feeling the need for the tonic to be preceded by the chord whose root is a perfect 5th higher.

By assigning degrees of the diatonic scale to the notes in the ordering of 5ths, designating C as degree 1, a sensible reason for Dubois' strong and weak triad designation is revealed.

Strong and Weak Degrees

**strong degrees are marked in blue, neutral in tan, and weak in green*

Degrees in the C Major Scale	$\hat{4}$	$\hat{1}$	$\hat{5}$	$\hat{2}$	$\hat{6}$	$\hat{3}$	$\hat{7}$
Ordering of the Diatonic Notes by 5ths	F	C	G	D	A	E	B

From this arrangement, we see that degrees $\hat{1}$, $\hat{4}$ and $\hat{5}$ comprise three degrees to the left of center, whereas degrees $\hat{3}$, $\hat{6}$ and $\hat{7}$ comprise the three degrees to the right of center, with degree $\hat{2}$ occupying a central position. Henceforth, as it is presented in the table above, I will identify degrees in order of 5ths rather than in order of diatonic note or degree. Therefore, I will reframe Dubois' strong and weak degrees so that the strong degrees are $\hat{4}$, $\hat{1}$ and $\hat{5}$, while the weak degrees are $\hat{6}$, $\hat{3}$ and $\hat{7}$. This will mean that, even if the triad is minor, because it is formed on a strong degree, it will sound stronger than even a major triad (VI) formed on any of the weaker degrees.

By extension, there are also STRONG and WEAK modes, the tonics of which correspond to the diatonic notes as shown above.

Strong and Weak Modes

Abbreviations: Lydian: LY, Ionian: IO, Mixolydian: MX, Dorian: DO, Aeolian: AE, Phrygian: PH, Locrian: LO

Degrees in the C Major Scale	$\hat{4}$	$\hat{1}$	$\hat{5}$	$\hat{2}$	$\hat{6}$	$\hat{3}$	$\hat{7}$
Ordering of the Diatonic Notes by 5ths	F	C	G	D	A	E	B
Corresponding Mode	LY	IO	MX	DO	AE	PH	LO

The modes formed upon F, C and G are of a different sonic nature than those formed upon A, E and B because of the primary di-chords which they contain. The Lydian, Ionian, and Mixolydian each contain di-chords [2] (major 2nd), [4] (major 3rd), [7] (perfect 5th) and [9] (major 6th) which cause these modes to sound generally more open and 'major'. By contrast, the Aeolian, Phrygian, and Locrian modes each contain di-chords [3] (minor 3rd), [5] (perfect 4th), [8] (minor 6th) and [10] (minor 7th), causing these modes to sound more contracted, tense, and 'minor'.

Strong Modes

	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$
Ploger-1st: Lydian	0	2	4	6	7	9	11	0
Ploger-2nd: Ionian	0	2	4	5	7	9	11	0
Ploger-3rd: Mixolydian	0	2	4	5	7	9	10	0

Weak Modes

	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$
Ploger-5th: Aeolian	0	2	3	5	7	8	10	0
Ploger-6th: Phrygian	0	1	3	5	7	8	10	0
Ploger-7th: Locrian	0	1	3	5	6	8	10	0

The Dorian as the fulcrum between the strong and weak modes

It might seem strange to modern musicians that Zarlino considered the Dorian the first mode, especially in light of the fact that this mode, founded originally on the note D, is in an innocuous location in the scale - degree 2. However, as we shall now see, in considering the ordering of the modes according to 5ths, the choice of Dorian as a first mode is understandable.

While the Lydian, Ionian, and Mixolydian are deemed strong and the Aeolian, Phrygian, and Locrian are deemed weak modes, the Dorian rests directly between both groups and acts as a sort of musical tipping point, sharing aspects of both the strong and weak modes. The Dorian shares three of the set of four primary di-chords that characterize the strong modes - [2, 7, 9] and also shares three of the set of four primary di-chords that characterize the weak modes - [3, 5, 10], so that, of the set of six total primary di-chords of any mode, there are an equal number of strong and an equal number of the weak, making the Dorian the most balanced in this regard.

Another aspect of balance expressed by the Dorian mode is seen by the fact that the same di-chords exist on either side of the tonic: There are notes both a [2] above and a [2] below the tonic; there are notes both a [3] above below the tonic; there are notes both a [5] above and below the tonic; there are notes both a [7] above and below the tonic; there are notes both a [9] above and below the tonic; and there are notes both a [10] above and below the tonic.

						D						
					C	D	E					
				B		D		F				
			A			D			G			
		G				D				A		
	F					D					B	
E						D						C
10	9	7	5	3	2	0	2	3	5	7	9	10

The note D, located at the center of the F-B ordering of the 5ths, acts as a type of mirror that reflects equally in both directions. This might explain why the Dorian might have been named the 1st mode by Zarlino and his predecessors and why so many composers, past and present, have a predilection for this mode, which expresses so many dimensions of equilibrium.

More patterns in the proposed ordering of the modes

Using the note ordering F-C-G-D-A-E-B as a basis for ordering the modes Lydian-Ionian-Mixolydian-Dorian-Aeolian-Phrygian-Locrian illuminates various other symmetries relating to the primary di-chords and therefore, relating to the underlying sonic characteristics of each mode.

Each mode consists of a tonic and six non-tonic notes that have been referred to as degrees $\hat{1}$, $\hat{2}$, $\hat{3}$, $\hat{4}$, $\hat{5}$, $\hat{6}$ and $\hat{7}$. However, in order to avoid confusion in the use of numbers that can mean either a degree or a di-chord, instead of numbers for degrees, I will now use the classic terms tonic (T), supertonic (SupT), mediant (M), subdominant (SubD), dominant (D), submediant (SubM) and subtonic (SubT), respectively.

$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$
T	SupT	M	SubD	D	SubM	SubT

The diatonic scale contains 42 di-chords:

1. seven diatonic 2nds
2. seven diatonic 3rds
3. seven diatonic 4ths
4. seven diatonic 5ths
5. seven diatonic 6ths
6. seven diatonic 7ths

Of the seven diatonic 2nds, five are major [2] and two are minor [1].

- [2] is found above the notes F-C-G-D-A, providing the SupT of the Lydian-Ionian-Mixolydian-Dorian-Aeolian modes (the first five modes in order by 5ths).

- [1] is found above the notes E-B, providing the SupT of the Phrygian and Locrian modes (the remaining two modes in order by 5ths).

Of the seven diatonic 3rds, three are major [4] and three are minor [3].

- [4] is found above the notes F-C-G, providing the M of the Lydian-Ionian-Mixolydian modes (the first three modes in order by 5ths).

- [3] is found above the notes D-A-E-B, providing the M of the Dorian-Aeolian-Phrygian-Locrian modes (the last four modes in order by 5ths).

Of the seven diatonic 4ths, only one is augmented [6], and the remaining six 4ths are perfect [5].

- [6] is found above the note F, providing the SubD of the Lydian mode. This is the only location where the SubD is [6] rather than [5].

- [5] is found above the notes C-G-D-A-E-B, providing the SubD of the Ionian-Mixolydian-Dorian-Aeolian-Phrygian-Locrian modes.

The principle of inversion

Knowing the placement of the 2nds, 3rds and 4ths, we can deduce the location of 7ths, 6ths and 5ths respectively because the latter will be inversions of the former. Therefore, the notes forming [1] will, when inverted, form its inversion [11]; the notes forming [2], when inverted, form the inversion [10]; et cetera.

Using the diatonic ordering by 5th that I am suggesting in this paper, we can see some delightful patterns related to the principle of inversion. For instance, the inversion of [1] is [11], the inversion being obtained by subtracting the first number from [12] - the octave.

Curiously, di-chords of the same size found in different locations in the scale will be perceived as different in sound - for example, in the Dorian mode, the A up to D will sound quite different B up to E, even though they are both perfect fourths (di-chord [5]).

- six [5]s; six [7]s

- five [2]s; five [10]s

- four [3]s; four [9]s

- three [4]s; three [8]s

- two [1]s; two [11]s

- one [6]

- While [1] is found only above the <u>last two</u> notes in the Pythagorean Ordering (B,E), [11] is found above the <u>first two</u> in that ordering (F, C).

- While [2] is found above the <u>first five</u> notes in the Pythagorean Ordering (F, C, G, D,A), [10] is found above the <u>last five</u> notes (B, E, A, D, G).

- While [3] is found above the <u>last four</u> notes in the Pythagorean Order (B, E, A, D), [9] is found above the <u>first four</u> notes in that order (F, C, G, D).

- While [4] is found only above the <u>first three</u> notes in the Pythagorean Order (F,C,G), [8] is found only above the <u>last three</u> notes in the Pythagorean Order (B, E, A).

- While [5] is found above the <u>last six</u> notes in the Pythagorean Order (B-E-A-D-G-C), [7] is found above the <u>first six</u> notes in the Pythagorean Order (F, C, G, D, A, E)

Location of di-chords and their inversions above each note in the diatonic scale

[1] and [11]

F	C	G	D	A	E	B
					1	1
11	11					

[2] and [10]

F	C	G	D	A	E	B
2	2	2	2	2		
		10	10	10	10	10

[3] and [9]

F	C	G	D	A	E	B
			3	3	3	3
9	9	9	9			

[4] and [8]

F	C	G	D	A	E	B
4	4	4				
				8	8	8

[5] and [7]

F	C	G	D	A	E	B
	5	5	5	5	5	5
7	7	7	7	7	7	

[6] and [6]

F	C	G	D	A	E	B
6						
						6

In observing the charts above, the number of occurrences of each di-chord and its inversion in the diatonic scale can seem random. I wondered if, by adding the sum of each of the two subcategories of di-chords (one the inversion of the other), there would be an underlying logic to the these sums.

For example, in the category that includes di-chords [1] and [11], there are 2 di-chord [1]s and, therefore, 2 di-chord [11]s that, when combined, add up to 24, expressed as the following equation: [2(1) + 2(11) = 24].

Similarly, the diatonic scale contains:

-5 of both di-chords [2] and [10], generating the equation [5(2) + 5(10) = 60]
-4 of both di-chords [3] and [9], generating the equation [4(3} + 4(9) = 48]
-3 of both di-chords [4] and [8], generating the equation [3(4) + 3(8) = 36]
-6 of both di-chords [5] and [7], generating the equation [6(5) + 6(7) = 72]
-2 di-chord [6]s, generating the equation [1(6) + 1(6) = 12].

The list of sums, in the order given above, is: 24, 60, 48, 36, 72 and 12. While it is evident that each of these sums is a multiple of 12, the ordering still seems too random. It finally occurred to me that, indeed, a pattern of perfect fifths underlays this random, zig-zagging arrangement of sums 24, 60, 48, 36, 72 and 12. I noticed that the largest sum - 72 - corresponds to those pitches closest by perfect fifths above [7] and below [5]the central or pivot note class [0], using the note F on the musical keyboard as [0].

Further, by extending yet again outward by perfect fifths, in both directions, both the di-chords [2] and [10] respectively, yield the next largest sum - 60. By again extending outward by perfect fifths, in both directions, both the di-chords [9] and [3] respectively, yield the next largest sum - 48. Continuing extending outward by perfect fifths, in both directions, both the di-chords [4] and [8] respectively, add up to the next largest sum - 36. Continuing again extending outward by perfect fifths, in both directions, both the di-chords [11] and [1] respectively, produce the next largest sum - 24. Finally, by again extending outward by perfect fifths, in both directions, both the di-chords [6] and [6], add up to the smallest sum - 12.

Diatonic scale mutation by 5ths

Below, I have listed the how the diatonic scale can be seen as evolving symmetrically upward and downward from the original F-C-G-D-A-E-B form, adding another note with a sharp a 5th higher, or adding another note with a flat a 5th lower, dropping off a note from the opposite end that appears at the other end of the line, maintaining seven notes in each line.

<u>Ploger Arrangement of Major Heptachords</u>

from the original, published 1986

C#															F#	C#	G#	D#	A#	E#	B#
F#														B	F#	C#	G#	D#	A#	E#	
B													E	B	F#	C#	G#	D#	A#		
E												A	E	B	F#	C#	G#	D#			
A											D	A	E	B	F#	C#	G#				
D										G	D	A	E	B	F#	C#					
G									C	G	D	A	E	B	F#						
C								**F**	**C**	**G**	**D**	**A**	**E**	**B**							
F							B♭	F	C	G	D	A	E								
B♭						E♭	B♭	F	C	G	D	A									
E♭					A♭	E♭	B♭	F	C	G	D										
A♭				D♭	A♭	E♭	B♭	F	C	G											
D♭			G♭	D♭	A♭	E♭	B♭	F	C												
G♭		C♭	G♭	D♭	A♭	E♭	B♭	F													
C♭	F♭	C♭	G♭	D♭	A♭	E♭	B♭														

*blue indicates the tonic pitch; the CM scale is highlighted in green

Conclusion

In this article, we have examined how a sequence of perfect fifths, starting on F and ending on B, might explain the seemingly random interval patterns found in diatonic scales and modes, and we have seen how fifths were respected by ancient theorists, starting with the

Chinese and Pythagoras and continuing through the present time. It is my hope that this examination might aid musicians in better understanding modes and, more importantly, that this study reveals the beauty and order permeating music. In the current era that is prone to mistrusting simple, basic principles, such revelations might inspire confidence in those seeking to understand the mysteries of music. Perhaps, looking below the surface of other seemingly random patterns, we may find order in other areas of our human experience.